The Arithmetic of Symmetry
Monday Mornings, 10-12, Dewey Hall
John Armstrong, moderator

(See end of this document for suggestions on presentation topics)

Symmetry is ubiquitous in nature, in art and music, and in mathematics. This seminar aims to deepen our appreciation for symmetry in all these areas.

Most **flora and fauna** are symmetric in some way. **Garden design** in some traditions is highly symmetrical. Some of **Bach's music** is explicitly symmetrical in time. **Ruby and sapphire** are different for the wearer, but their crystal symmetry is the same; **diamond's symmetry** is different.

Chiral symmetry (also called mirror symmetry) is exemplified by the difference between the left hand and and the right. It is a remarkable fact that many biologically crucial molecules have both left-handed and right-handed varieties, but **only the left-handed occur naturally in living systems.**

Floor tilings exhibit a dizzying array of symmetries. But if the individual tiles are identical regular polygons, the number of **possible** tessellations is precisely **17**. **M. Escher**'s art is often based on exploitation of symmetry, sometimes with subtle, tantalizing **violations** of that symmetry. The human body is '**almost symmetric**.' Another example of 'almost symmetric' is the difference between **baroque** interior decoration and its **rococo** successor. The 'almost symmetric' is a very powerful concept, both in art and in science.

It is a remarkable fact that these diverse forms of symmetry have an **underlying, unifying**, mathematical structure...the theory of **finite groups**. The theory is not difficult, but it will be new to most LIR members. It teaches us two new types of multiplication. One is the 'multiplication of symmetry operations', in which X times Y need not equal Y times X. The symmetries of the equilateral triangle offer the simplest example!

Fractals are mathematical objects whose symmetry is '**self-similarity**'. Here, too, the underlying math is addition plus a type of enhanced multiplication... the multiplication of complex numbers. If you can add, and can expand your concept of multiplication, a new appreciation of symmetry will reward you.

When an element of **randomness** is present, fractals characterize many seemingly non-symmetric, but in fact statistically **self-similar**, natural phenomena: e.g. clouds, or coastlines, or mountains, or records of river flooding, or the (natural ??) time behavior of **financial markets.**

For a good working definition of symmetry, useful in this seminar, see
http://encyclopedia2.thefreedictionary.com/symmetry

For a **wonderful overview**, including **symmetry in a Bach Fugue**, see
https://www.youtube.com/watch?v=V5tUM5aLHPA

For more on the connection between math and music, also see:
https://www.youtube.com/watch?v=T-pew8_aBxA

On the connection between **fractals and observed phenomena**, see:
http://users.math.yale.edu/public_html/People/frame/Fractals/

Possible Presentation Topics

*(Feel free to discuss these, **or other** possible topics with the moderator)*

There is much symmetry in Indian, Persian and Moorish architecture; how many different symmetries from these cultures do we know? How do these symmetries contrast with those in Greek and Roman culture? Show examples.

Many cultures have produced Mosaics; what symmetries do they exhibit? Are there constraints on the possible forms? Floor tilings also exhibit many symmetries, but there also are constraints on the possibilities. Show examples.

Present on the 'Koch Snowflake', or the 'Serpinski Gasket' or other such 'self-similar' figures. The study of fractal, and/or self-similar, objects has been hugely stimulated by the availability of computers to do the massive amount of repetitive, arithmetic computation (iteration) involved. But there were self-similar objects known long before modern computers.

Plato was fascinated by the five 'Platonic solids'; what are they?; What are their symmetries? What roles have they played in western art, and philosophy, and science?

Traffic signs and other 'signs exhibit symmetry; how many can you find? What about 'emojis' or other 'folk signs'? How does the presence or absence of lettering affect the symmetries? What are the least and the most symmetric examples of public signage?

Snow flakes often exhibit hexagonal symmetry; what do we know about why this is?

Flowers are very often symmetric; what degrees of symmetry have botanists observed? Give examples of different floral symmetries. What role does the 'Fibonacci series' in floral symmetry?

Different cultures embody symmetry in art and architecture differently; many of these symmetries are 'simple' (from the perspective of possible symmetries). Are there cultures that embody more complex symmetries? Which ones?

Much of what we know about the properties of materials, including biologically important ones, comes from the study of their crystal symmetries. Describe the different ways in which crystal symmetries have been explored over the past several centuries.

Beauty (in any sense you care to consider) is usually related to symmetry. Are there beautiful natural objects, or beautiful human creations which do not exhibit symmetry?

Present on Amy Noether, who was a mathematician, a physicist, a woman, and a Jew in pre-World War II Germany; one of her contributions was "Amy Noether's Theorem", a landmark in understanding the connection between symmetry and the fundamental laws of nature.

Report on the ‘sport’ (or ‘science’) of cathedral bell ringing, which is entirely based on a certain type of symmetry.

Many musical compositions involve musical elements symmetric in time. Discuss, and give examples.